Theorems of the relative primes

Remark: In this article it is spoken only about the natural numbers. M! – M is factorial.

Theorem 1. The prime divisor of the numbers (2^n - 1), where n is simple, has the following aspect: An + 1, where A – is a natural number.

Proof. Let's compare the numbers (2^n – 1) and (2^f – 1), where f = M!/n^t, M – is infinitely large natural number, t – is infinitely large natural number, where the condition of (n,M!/n^t) = 1, is satisfied, so the numbers n and M!/n^t are relative primes, f – is a natural number. Because of M – is infinitely large natural number, for any prime p (with the exception of some of cases, when p - 1 = Bn, because f and n are relative primes. B – is a natural number) the following condition is satisfied:

$(2^f - 1) \equiv 0 \pmod{(2^{p-1} - 1)}$.

According to the Ferma's Small theorem it is known that

$(2^{p-1} - 1) \equiv 0 \pmod{p}$.

So, $(2^f - 1) \equiv 0 \pmod{p}$.

We know that (see «Theorems of relative primes numbers» on the site: http://logman-logman.narod.ru/) the numbers (2^n - 1) and (2^f - 1) are relative primes, because (n,f) = 1. It means that any prime is a divider of the number (2^f - 1), with the exception of some of cases, when p – 1 = Bn and p = Bn + 1. So, the prime divisor of the numbers (2^n - 1) have the following aspect: Bn + 1.

Bn + 1 = An + 1. The theorem is proved.

Theorem 2. The prime divisor of the numbers (S^n - 1)/(S – 1), where n is a prime , have the following aspect: (Tn + 1), and for the limit quotation of significances of n when S = constant the numbers (S^n - 1)/(S – 1) can have the prime divisor d, where T – is a natural number, d – is a number's (S – 1) divider.

Proof. Let's prove «Theorem 2» by the same way like we've done according to the "theorem 1". Let's compare the numbers (S^n - 1)/(S – 1) and (S^f - 1)/(S – 1), where f = M!/n^t, M – is infinitely large natural number, t – is infinitely large natural number, where the following condition is satisfied: (n,M!/n^t) = 1, so the numbers n and M!/n^t are relative primes, f – is a natural number:

a) So, we get the analogical result to the "theorem 1", and the prime divisor of numbers (S^n - 1)/(S – 1) have the following aspect: (Tn + 1). More over,

б) Because at the denominator of the number (S^f - 1)/(S – 1) there is a number (S – 1), there are some of cases, when the number (S^f - 1)/(S – 1) doesn't have any prime divisor d, which is the prime divisor of the number (S – 1). Because of this reason it is possible to suggest that such numbers like d may turned out to be the prime divisor of numbers (S^n - 1)/(S – 1). There are some of the similar cases. For example: $(10^3 – 1)/(10 – 1) \equiv 0 \pmod{3}$

In this way it is obvious that the quantity of significances of n is limited within quantity of the prime divisor of the number (S – 1) from above.

The theorem is proved.